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Unified Development of Lateral-Directional Departure Criteria

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Several frequently used departure prediction indicators for both open- and closed-loop control of flight are developed using a unified, rigorous analytical approach applied to a linear version of the aircraft model. These criteria are for departure caused by aerodynamic disturbances only. It is shown that these indicators are limited in their accuracy because of restrictive assumptions and terms omitted. A second approach is presented that leads to the same results as the first, but is more applicable to the nonlinear problem. Some ideas concerning the application of the linear methods to the nonlinear problem are presented.

Nomenclature

A	= system matrix, Eq. (1)
a_{ij}	= i, jth element of system matrix
a, b, c, d, e	= coefficients of characteristic equation
B	= control matrix, Eq. (1)
b	= wing span, ft
C_l, C_n, C_Y	= roll, yaw moment, and sideforce nondimensional coefficients
$C_{n_{\beta_{dyn}}}$	= lateral departure parameter, Eq. (17)
$C_{n_{\beta_{apparent}}}$	= closed-loop lateral departure parameter, Eq. (33)
g	= acceleration because of gravity, ft/s^2
I	= $I_x I_z - (I_{xz})^2$, slug-ft^2
I_n	= $n \times n$ identity matrix
$I_x I_y, I_z, I_{xz}$	= moments of inertia about the x, y, z axes, xz product of inertia, slug-ft^2
L	= dimensional roll moment, ft-lb
N	= dimensional yaw moment, ft-lb
p, q, r	= roll, pitch, and yaw rate about generic body x, y, and z axes, rad/s
$\bar{q}$	= dynamic pressure, psf
S	= reference wing planform area, ft^2
V	= airspeed, ft/s
Y	= dimensional side force, lb
α_b	= angle-of-attack measured with respect to generic body x axis
β	= sideslip angle
$\Delta(\cdot)$	= perturbation value of $(\cdot)$
θ	= pitch-attitude angle
λ	= eigenvalue associated with system matrix
ϕ	= bank-attitude angle

Subscript

x	= partial derivative of $(\cdot)$ with respect to x, where x can be β, p, or r

Superscripts

s	= quantity $(\cdot)$ evaluated in stability axes
$(\cdot)'$	= dimensional or nondimensional stability derivatives including moment of inertia terms, Eqs. (7) and (14)
$(\cdot)$	= quantities with no superscript are evaluated in generic body axes

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Introduction

ALTHOUGH current fighter aircraft are generally limited to low angles of attack (AOA), future aircraft designed for an air superiority role will be required to have high AOA maneuvering capabilities.[1–5] It is, therefore, necessary that the aircraft have sufficient inherent stability or that it provide sufficient control power to prevent departure from controlled flight. Sufficient control power is required so that departure resistance can be maintained by appropriate pilot inputs or by using feedback stability augmentation methods. In either case, some design criteria must be available to characterize departure behavior.

Although aircraft can depart in many different ways, most of the existing departure criteria are related to lateral-directional modes characterized by a nose-slice, a wing-drop, or a wing-rock motion. These departure modes can be caused by several different mechanisms, asymmetric wing stall or unstall (wing drop),[6] aerodynamic yaw moments exceeding the control authority of the rudder (nose slice),[7,8] and aerodynamic rate-damping moments becoming negative (wing rock).

Attempts to develop criteria to be used early in the design process have been made over the past several years.[9–12] The early philosophies were developed prior to the computer age and thus aimed at simple back-of-the-envelope calculations that lead to criteria easily evaluated in the early design stages. Until recently, this philosophy has not changed. One of the reasons for this lack of change is the success of these early criteria when applied to rather conventional aircraft in predicting departure resistance. With the recent interest in high AOA maneuvering and the new unconventional fighter aircraft configurations, however, these simple methods have become less accurate in predicting departure susceptibility.[13] A recent survey of many of the currently used and proposed criteria is presented by Seltzer.[14]

Although the early criteria were based on heuristic arguments coupled with analyzing a considerable amount of data, these, along with more recent criteria, can be developed rigorously from basic principles and put on a sound theoretical foundation.[15,16] Basically there are two approaches that are used to develop such criteria, both of which start with the vehicle equations of motion. The first approach is to linearize these equations of motion about some reference flight condition, and then do some linear system analysis on the resulting equations,[9,11,13] whereas the second approach deals directly with the equations of motion with particular attention given to angular accelerations about appropriate axes.[15,16] The latter approach has been used recently to include coupling effects observed in dynamic maneuvers in the derived departure criteria.[17,18] Unfortunately in many of the earlier developments and subsequent applications, rigor is not retained throughout and confusion arises as to what reference systems are used and how the aerodynamic stability derivatives are defined. This paper shows that the most frequently used criteria for both open- and closed-loop flight can be developed from a unified approach. It also shows that most current criteria are subject to severe limiting assumptions with regard to terms ignored.

Open-Loop Stability Criteria

In general, the conditions for departure resistance are based on one of two considerations, static stability or dynamic stability. In each case we must start with a reference flight condition. Typically for departure considerations, this condition is symmetric or near symmetric flight at a high AOA. The static stability criterion asks the question, if disturbed from this reference flight condition, does the disturbance cause forces and/or moments that oppose the disturbance? On the other hand, the dynamic stability criterion asks the question, does the system return to the reference flight condition in time? Hence, static stability requires the forces and moments resulting from disturbances to be in the correct direction, whereas dynamic stability requires additionally that they be in the correct phase. Static stability considerations can be developed from knowledge of only aerodynamic characteristics, whereas dynamic stability considerations require knowledge of the complete dynamic system.

Static stability requirements can be obtained by determining the change in a force or moment resulting from a change in some disturbance variable. The result is usually in the form of a condition on some stability derivative such as $N_\beta > 0$. However, care must be taken in interpreting the result.

Dynamic stability requirements are well defined for linear systems of the form

$$\Delta\dot{x} = A\Delta x + B\Delta u \tag{1}$$

where Δx is the perturbed state vector and Δu the perturbed control vector. For the lateral-directional dynamics of an aircraft, the linear dynamic system has four states of interest,

$$\Delta x^T = [\Delta\beta, \Delta p, \Delta r, \Delta\phi] \tag{2}$$

A necessary and sufficient condition for dynamic stability is that the roots (eigenvalues) of the characteristic equation

$$|\lambda I_4 - A| = 0 \tag{3}$$

have negative real parts. This characteristic equation has the generic form

$$a\lambda^4 + b\lambda^3 + c\lambda^2 + d\lambda + e = 0 \tag{4}$$

Although an analytic solution to Eq. (4) is available, it is not easily written in a useful literal form. An early mathematician by the name of Routh, however, determined criteria that must be satisfied if the roots to a polynomial are to have negative real parts.[19] For a fourth-order equation these criteria are

$$\begin{gathered} a, b, c, d, e > 0; \qquad bc - ad > 0 \\ d(bc - ad) - b^2 e > 0 \end{gathered} \tag{5}$$

For a third-order system it is required that all of the coefficients be positive, as Eq. (5), and that $(bc - ad) > 0$, and for a second-order system the only requirement is that all of the coefficients be positive. In all cases the condition that the last coefficient be greater than zero (for the quartic, $e > 0$) is sometimes considered as the generalized static stability criterion for the complete system because when it changes sign a real root with a positive real part will appear. Hence, this criterion defines more precise requirements for static stability than the simple derivative idea mentioned previously.[19] It is this last result that is used to derive the various departure criteria of interest.

Lateral-Directional Linear Stability

It is of interest to examine the generalized static stability term for the lateral-directional motion to give additional insight into the departure problem. We linearize the equations of motion about a symmetric ($\beta = 0$), steady-state flight condition. Under these circumstances the lateral-directional system matrix is given by[13,19]

$$A = \begin{bmatrix} Y'_\beta & (Y'_p/V) + \sin\alpha_b & (Y'_r/V) - \cos\alpha_b & g\cos\theta/V \\ L'_\beta & L'_p & L'_r & 0 \\ N'_\beta & N'_p & N'_r & 0 \\ 0 & 1 & \tan\theta & 0 \end{bmatrix}_{\text{ref}} \tag{6}$$

The subscript (ref) indicates that the quantities in the matrix are evaluated at the reference flight condition of interest. The primed quantities are defined as

$$\begin{aligned} Y'_{(\cdot)} &= \frac{1}{m}\frac{\partial Y}{\partial(\cdot)} \\ L'_{(\cdot)} &= (1/I)\big[I_z L_{(\cdot)} + I_{xz} N_{(\cdot)}\big] \\ N'_{(\cdot)} &= (1/I)\big[I_{xz} L_{(\cdot)} + I_x N_{(\cdot)}\big] \end{aligned} \tag{7}$$

Note that the aerodynamic moments, stability derivatives, and inertia properties are dependent on the axis system selected.

The characteristic equation of the matrix in Eq. (6) has its coefficients given in terms of the matrix elements a_{ij} by

$$\begin{gathered} a = 1, \qquad b = -(a_{11} + a_{22} + a_{33}) \\ c = a_{11}a_{22} + a_{11}a_{33} + a_{22}a_{33} - a_{12}a_{21} - a_{13}a_{31} - a_{32}a_{23} \\ d = a_{11}(a_{32}a_{23} - a_{22}a_{33}) + a_{12}(a_{21}a_{33} - a_{31}a_{23}) \\ + a_{13}(a_{31}a_{22} - a_{21}a_{32}) - a_{14}(a_{21} + a_{31}a_{43}) \\ e = a_{14}[(a_{21}a_{33} - a_{31}a_{23}) + a_{43}(a_{31}a_{22} - a_{21}a_{32})] \end{gathered} \tag{8}$$

The generalized static stability criterion for lateral-directional motion is given by $e > 0$, or

$$(g/V)\,[(L'_\beta N'_r - N'_\beta L'_r)\cos\theta + (N'_\beta L'_p - L'_\beta N'_p)\sin\theta] > 0 \tag{9}$$

Also note that the condition $e > 0$ holds for all coordinate systems provided that all of the aerodynamic stability derivatives and inertia terms are calculated in the same coordinate system.

It can be observed from the condition given by Eq. (9) that gravity is an important contributor. If gravity were neglected, this term would vanish. We will assert that gravity should not directly cause a lateral-directional aerodynamic instability. Hence, if we restrict our interest to departure resulting from aerodynamic causes, we can neglect the gravity term and, thus, any results obtained will be those resulting solely from aerodynamic disturbances.

The basic static stability conditions require that the perturbations in the state variables develop forces or moments that oppose the perturbations. The objective here is to show that if we include only force and moment terms resulting from displacements in the system matrix, Eq. (6), the generalized static stability requirement will lead to the classic $C_{n_{\beta_{\text{dyn}}}} > 0$ requirement. The approach taken here is similar to that used by Calico.[11] The fundamental idea is that the aerodynamic forces and moments resulting from angular displacements must provide the stability for departure prevention. Hence, if we keep only these terms and neglect all others (including gravity), we have

$$A = \begin{bmatrix} Y'_\beta & \sin\alpha_b & -\cos\alpha_b & 0 \\ L'_\beta & 0 & 0 & 0 \\ N'_\beta & 0 & 0 & 0 \\ 0 & 1 & \tan\theta & 0 \end{bmatrix}_{\text{ref}} \tag{10}$$

The characteristic equation becomes

$$\lambda^2\big[\lambda^2 - Y'_\beta\lambda + N'_\beta\cos\alpha_b - L'_\beta\sin\alpha_b\big] = 0 \tag{11}$$

Here we have lost two roots, one resulting from the assumption of no gravity and the other resulting from the assumption of no damping terms (the spiral and rolling convergence modes, respectively). The requirements due to Routh for stability of the roots of the remaining quadratic equation are

$$Y'_\beta < 0, \qquad N'_\beta\cos\alpha_b - L'_\beta\sin\alpha_b > 0 \tag{12}$$

As already observed, Eq. (12) holds for all axis systems. Hence, we can select the stability axes system ($\alpha_b = 0$) and write the last condition as

$$(N'_\beta)^s > 0 \tag{13}$$

where the superscript s refers to the body-fixed stability axes. If we define the terms

$$C'_{n_\beta} = \frac{[C_{n_\beta} + (I_{xz}/I_x)C_{l_\beta}]}{[1 - (I_{xz}^2/I_xI_z)]} \quad C'_{l_\beta} = \frac{[C_{l_\beta} + (I_{xz}/I_z)C_{n_\beta}]}{[1 - (I_{xz}^2/I_xI_z)]} \tag{14}$$

Equations (7), (12), and (13) can be reduced to the equivalent condition

$$\begin{aligned}(1/I_z^s)C_{n_\beta}^{s'} &= (1/I_z)C'_{n_\beta}\cos\alpha_b - (1/I_x)C'_{l_\beta}\sin\alpha_b > 0 \\ (I_z/I_z^s)C_{n_\beta}^{s'} &= C'_{n_\beta}\cos\alpha_b - (I_z/I_x)C'_{l_\beta}\sin\alpha_b > 0 \\ &\equiv C_{n_{\beta_{\rm dyn}}} > 0\end{aligned} \tag{15}$$

where the absence of a superscript refers to an arbitrarily selected body-fixed axes system. In terms of the basic stability derivatives we have

$$\begin{aligned}C_{n_{\beta_{\rm dyn}}} &= \left[\frac{C_{n_\beta} + (I_{xz}/I_x)C_{l_\beta}}{1 - (I_{xz}^2/I_xI_z)}\right]\cos\alpha_b \\ &- \frac{I_z}{I_x}\left[\frac{C_{l_\beta} + (I_{xz}/I_z)C_{n_\beta}}{1 - (I_{xz}^2/I_xI_z)}\right]\sin\alpha_b\end{aligned} \tag{16}$$

$$C_{n_{\beta_{\rm dyn}}} = \frac{I_z}{I_z^s}\left[\frac{C_{n_\beta} + (I_{xz}/I_x)C_{l_\beta}}{1 - (I_{xz}^2/I_xI_z)}\right]^s \tag{17}$$

For the special case of principal axes ($I_{xz} = 0$), these results reduce to those originally proposed by Moul and Paulson.[9] For the reduced system given by Eq. (10), $C_{n_{\beta_{\rm dyn}}}$ is directly related to the generalized static stability requirement. It also indicates that the standard lateral-directional stability parameter criterion $C_{n_\beta} > 0$ is valid only if evaluated in stability axes!

An alternative approach similar to that suggested by Pelikan[16] and related to the approach of Kalviste and Eller[18] that can be applied to the complete nonlinear system is presented next. It will be applied to the reduced linear system of equations for comparative purposes. If we select generic body-fixed axes, we can write the following relations from Eqs. (1), (2), and (10):

$$\Delta\dot p = (L'_\beta)\Delta\beta, \qquad \Delta\dot r = (N'_\beta)\Delta\beta \tag{18}$$

Transformation to stability axes and the introduction of Eq. (18) leads to the result

$$\begin{aligned}\Delta\dot r^s &= -\Delta\dot p\sin\alpha_b + \Delta\dot r\cos\alpha_b \\ &= (-L'_\beta\sin\alpha_b + N'_\beta\cos\alpha_b)\Delta\beta \\ &= N_\beta^{s'}\Delta\beta\end{aligned} \tag{19}$$

Using Eq. (19) along with the static stability requirement of having an angular acceleration (moment) to oppose a disturbance in the sideslip angle, and noting that the sideslip angle is measured about the stability z axis, we have

$$\frac{\partial\Delta\dot r^s}{\partial\beta} = \frac{\bar q Sb}{I_z^s}C_{n_\beta}^{s'} = \frac{\bar q Sb}{I_z}C_{n_{\beta_{\rm dyn}}} > 0 \tag{20}$$

This result is the same as that developed earlier in Eq. (15) and validates this approach as a method to predict lateral-directional departure. It is suitable for the complete nonlinear dynamic system but a discussion of that digresses from the objectives of this paper.

From the preceding remarks one can see that $C_{n_{\beta_{\rm dyn}}} > 0$ is really a static stability criterion and is obtained in its pure form only by making several restrictive assumptions. The information lost because of assumptions, however, can cause inaccurate results. Attempts have been made to reduce these assumptions but still retain tractable results.[13] We will eliminate only the gravity term, as indicated earlier. Under these circumstances the system reduces to a system with a third-order characteristic equation. The system matrix becomes

$$A = \begin{bmatrix} Y'_\beta & (Y'_p/V) + \sin\alpha_b & (Y'_r/V) - \cos\alpha_b & 0 \\ L'_\beta & L'_p & L'_r & 0 \\ N'_\beta & N'_p & N'_r & 0 \\ 0 & 1 & \tan\theta & 0 \end{bmatrix}_{\rm ref} \tag{21}$$

The associated characteristic equation is a quartic with one zero root,

$$\lambda[a\lambda^3 + b\lambda^2 + c\lambda + d] = 0 \tag{22}$$

where

$$\begin{aligned} a &= 1, \qquad b = -(y'_\beta + L'_p + N'_r) \\ c &= Y'_\beta(L'_p + N'_r) + L'_pN'_r - N'_pL'_r \\ &\quad - [(Y'_p/V) + \sin\alpha_b]L'_\beta - [(Y'_r/V) - \cos\alpha_b]N'_\beta \\ d &= [(Y'_p/V) + \sin\alpha_b](L'_\beta N'_r - N'_\beta L'_r) + Y'_\beta(N'_pL'_r - L'_pN'_r) \\ &\quad + [(Y'_r/V) - \cos\alpha_b](N'_\beta L'_p - L'_\beta N'_p)\end{aligned} \tag{23}$$

The conditions for stability are that $b, c,$ and $d > 0$ and that $bd - c > 0$. The requirement that $c > 0$ leads to terms that include $C_{n_{\beta_{\rm dyn}}}$. In particular, we have

$$\begin{aligned} N'_\beta\cos\alpha_b - L'_\beta\sin\alpha_b &> -Y'_\beta(L'_p + N'_r) - L'_pN'_r \\ &\quad + N'_pL'_r + (Y'_p/V)L'_\beta + (Y'_r/V)N'_\beta\end{aligned} \tag{24}$$

Hence, a lower bound on $C_{n_{\beta_{\rm dyn}}}$ is determined that is related to the earlier ignored rate terms. Similar results are presented in Ref. 13. Additional criteria can be determined from the other conditions that b and $d > 0$ and $bc - d > 0$. In each case the equations can be expanded in terms of the basic stability derivatives C_{n_β}, C_{l_β}, C_{n_p}, C_{n_r}, etc., and the moments and products of inertia. The forms of the resulting equations are quite complicated but are easily implemented on a computer.

Closed-Loop Departure Criteria

The techniques of the preceding section can be extended to the closed-loop controlled case in a straightforward manner. Consider the case of full state feedback with only aileron and rudder as controls. In this case the feedback control law takes the form

$$\begin{Bmatrix}\Delta\delta_a \\ \Delta\delta_r\end{Bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24}\end{bmatrix}\begin{Bmatrix}\delta\beta \\ \Delta p \\ \Delta r \\ \Delta\phi\end{Bmatrix} \tag{25}$$

or

$$\Delta\boldsymbol{u} = K\Delta\boldsymbol{x} \tag{26}$$

With this control law substituted into Eq. (1), the closed-loop controlled system is given by

$$\Delta\dot{\boldsymbol{x}} = A_{\rm cl}\Delta\boldsymbol{x} \tag{27}$$

where

$$A_{\rm cl} = A + BK \tag{28}$$

Yaw Stability

We can parallel the development in the preceding section and consider first the case where the rate terms (and gravity) are ignored or assumed zero. Also we will assume only a rudder-sideslip angle feedback gain, all others being zero. We can, therefore, designate the gain $k_{21} = k$. Under these conditions the augmented system matrix becomes

$$A_{cl} = \begin{bmatrix} Y'_\beta + kY'_{\delta_r} & \sin\alpha_b & -\cos\alpha_b & 0 \\ L'_\beta + kL'_{\delta_r} & 0 & 0 & 0 \\ N'_\beta + kN'_{\delta_r} & 0 & 0 & 0 \\ 0 & 1 & \tan\theta & 0 \end{bmatrix}_{\text{ref}} \tag{29}$$

With suitable definitions of primed control derivatives [see Eqs. (7) and (14)], Routh's criteria for the reduced system become

$$Y'_\beta + kY'_{\delta_r} < 0 \tag{30}$$

and

$$-\left(L'_\beta + kL'_{\delta_r}\right)\sin\alpha_b + \left(N'_\beta + kN'_{\delta_r}\right)\cos\alpha_b > 0 \tag{31}$$

This last equation leads to the result

$$\begin{aligned}(I_z/I_z^s)\left(C'_{n_\beta} + kC'_{n_{\delta_r}}\right)^s &= \left(C'_{n_\beta} + kC'_{n_{\delta_r}}\right)\cos\alpha_b \\ &- (I_z/I_x)\left(C'_{l_\beta} + kC'_{l_{\delta_r}}\right)\sin\alpha_b > 0\end{aligned} \tag{32}$$

Define

$$\begin{aligned}C_{n_{\beta_\text{apparent}}} &= \left(C'_{n_\beta} + kC'_{n_{\delta_r}}\right)\cos\alpha_b - (I_z/I_x) \\ &\times \left(C'_{l_\beta} + kC'_{l_{\delta_r}}\right)\sin\alpha_b > 0\end{aligned} \tag{33}$$

Equation (33) is the closed-loop departure criterion for the case where k is the sideslip-rudder gain for the aircraft.

The results are determined from the linearized equations of motion and all of the derivatives are evaluated at the reference flight condition. Since aerodynamic terms become highly nonlinear at high AOA and local slopes required for the derivatives may not be representative of the global behavior, alternative approximations to these derivatives can be used. Pelikan[16] suggests using sideslip related derivatives evaluated using the secant method. Here, the derivative is determined as the value of the force or moment coefficient divided by the sideslip angle. Using this definition, we can now show that the result in Eq. (33) leads to the same result obtained by Pelikan.[16]

First observe that since the reference flight condition is at zero sideslip angle the perturbation value of the sideslip angle is equal to the sideslip angle and they can be used interchangeably, that is, $\Delta\beta = \beta$. We note that the gain term is

$$k = \Delta\delta_r/\Delta\beta = \Delta\delta_r/\beta \tag{34}$$

Substituting this value into Eq. (33) yields the typical term

$$\begin{aligned}C'_{n_\beta} + kC'_{n_{\delta_r}} &= C'_{n_\beta} + \frac{C'_{n_{\delta_r}}\Delta\delta_r}{\Delta\beta} \\ &= \frac{C'_{n_\beta}\Delta\beta + C'_{n_{\delta_r}}\Delta\delta_r}{\Delta\beta} \\ &= \frac{C'_n(\beta) + C'_n(\delta_r)}{\beta}\end{aligned} \tag{35}$$

The last line is the nonlinear equivalent of the linear results on the previous line. $C'_n(\beta)$ is the total yaw-moment coefficient for a given sideslip angle with no rudder control, $C'_n(\delta_r)$ is the total yaw moment coefficient resulting from rudder control deflection, and β is the total sideslip angle. Hence, the linear result is converted back to a form useful for nonlinear analysis with the two terms being the secant derivatives. If these results are substituted into Eq. (33), the resulting expression for the $C_{n_{\beta_\text{apparent}}}$ term is given by

$$\begin{aligned}C_{n_{\beta_\text{apparent}}} &= \left[\frac{C'_n(\beta) + C'_n(\delta_r)}{\beta}\right]\cos\alpha_b \\ &- \frac{I_z}{I_x}\left(\frac{C'_l(\beta) + C'_l(\delta_r)}{\beta}\right)\sin\alpha_b > 0\end{aligned} \tag{36}$$

Equation (36) is the equation developed by Pelikan[16] using the angular acceleration requirement, $(\partial\Delta\dot{r}^s/\partial\Delta\beta > 0)$, described earlier.

Roll Stability (Lateral Control Departure Parameter)

Here we will consider the case where all of the elements in the gain matrix, Eq. (25), except $k_{14} \equiv k$ are equal to zero. Under these circumstances the reduced augmented system (no rate or gravity terms) becomes

$$A_{cl} = \begin{bmatrix} Y'_\beta & \sin\alpha_b & -\cos\alpha_b & kY'_{\delta_a} \\ L'_\beta & 0 & 0 & kL'_{\delta_a} \\ N'_\beta & 0 & 0 & kN'_{\delta_a} \\ 0 & 1 & \tan\theta & 0 \end{bmatrix}_{\text{ref}} \tag{37}$$

The characteristic equation is a quartic with the coefficients

$$\begin{aligned}a &= 1, \qquad b = -Y'_\beta \\ c &= -k\left(N'_{\delta_a}\tan\theta + L'_{\delta_a}\right) - (L'_\beta\sin\alpha_b + N'_\beta\cos\alpha_b) \\ d &= -kY'_{\delta_a}(N'_\beta\tan\theta + L'_\beta) + kY'_\beta\left(N'_{\delta_a}\tan\theta + L'_{\delta_a}\right) \\ e &= -k\left(N'_\beta L'_{\delta_a} - L'_\beta N'_{\delta_a}\right)(\tan\theta\sin\alpha_b + \cos\alpha_b)\end{aligned} \tag{38}$$

For stability we require all of the coefficients to be positive. The generalized static stability requirements is that $e > 0$. Assuming the magnitude of the pitch attitude and AOA are less than 90 deg, this requirement reduces to

$$-k\left(N'_\beta L'_{\delta_a} - L'_\beta N'_{\delta_a}\right) > 0 \tag{39}$$

Equation (39) is the required stability criterion and can be shown to reduce to the lateral control departure parameter (LCDP), which is currently used. If it is assumed that the product of kL'_{δ_a} is negative, so that a positive roll angle would cause a negative rolling moment resulting from aileron control, Eq. (39) can be rearranged in the form

$$N'_\beta - L'_\beta\left(N'_{\delta_a}/L'_{\delta_a}\right) > 0 \tag{40}$$

In terms of the nondimensional stability derivatives, Eq. (40) becomes

$$\begin{aligned}&\left(C_{n_\beta} + \frac{I_{xz}}{I_x}C_{l_\beta}\right) - \left(C_{l_\beta} + \frac{I_{xz}}{I_z}C_{n_\beta}\right) \\ &\times \left[\frac{C_{n_{\delta_a}} + (I_{xz}/I_x)C_{l_{\delta_a}}}{C_{l_{\delta_a}} + (I_{xz}/I_z)C_{n_{\delta_a}}}\right] > 0\end{aligned} \tag{41}$$

Equations (40) and (41) are versions of the LCDP.[14,15] Note that the same results are obtained if we assume only roll-rate feedback to aileron, $k_{12} \neq 0$.

Aileron Rudder Interconnect

The stability of a system that includes an aileron–rudder interconnect (ARI) can be determined in a similar manner by including the gains k_{14} and k_{24} in the gain matrix Eq. (26). Here we have

$$\Delta\delta_a = k_{14}\Delta\phi = k_1\Delta\phi, \qquad \Delta\delta_r = k_{24}\Delta\phi = k_2\Delta\phi \tag{42}$$

$$\Delta\delta_r/\Delta\delta_a = k_2/k_1 = k \tag{43}$$

where k is ARI gearing ratio. Under these conditions the extra feedback control terms enter into the fourth column of the system matrix, with these terms becoming

$$\begin{aligned}a_{14} &= k_1Y'_{\delta_a} + k_2Y'_{\delta_r}, \qquad a_{24} = k_1L'_{\delta_a} + k_2L'_{\delta_r} \\ a_{34} &= k_1N'_{\delta_a} + k_2N'_{\delta_r}, \qquad a_{44} = 0\end{aligned} \tag{44}$$

All of the coefficients of the characteristic equation are changed somewhat because of the addition of the bank-angle feedback to rudder. The term of interest, however, is the last coefficient, e. With the same assumptions as in the preceding section, the interesting part of this term becomes

$$-\left\{\left(k_1 L'_{\delta_a} + k_2 L'_{\delta_r}\right) N'_\beta - \left(k_1 N'_{\delta_a} + k_2 N'_{\delta_r}\right) L'_\beta\right\} > 0 \qquad (45)$$

Again if we assume the coefficient of the N'_β is negative, we can rearrange the terms in Eq. (45) to put the equation in the desired form,

$$N'_\beta - L'_\beta \left(\frac{N'_{\delta_a} + kN'_{\delta_r}}{L'_{\delta_a} + kL'_{\delta_r}}\right) > 0 \qquad (46)$$

or

$$N'_\beta - L'_\beta\left(N'_{\delta_a}/L'_{\delta_a}\right) + k\left[N'_\beta - L'_\beta\left(N'_{\delta_r}/L'_{\delta_r}\right)\right]\left(L'_{\delta_r}/L'_{\delta_a}\right) > 0 \quad (47)$$

where $k = \Delta\delta_r/\Delta\delta_a$, as defined earlier in Eq. (43). Note that the same results are obtained assuming only roll-rate feedback to aileron and rudder.

These results are the same as those found in Refs. 9 and 15, where they were found using an approach completely different from that used here.

Summary and Conclusions

As originally stated, the purpose of this paper is to show that the most frequently used departure predicting parameters can be derived from a unified approach. This approach consists of linearizing the equations of motion, making suitable assumptions, and examining the stability of the resulting dynamic system using some of Routh's criteria. In particular, the criterion used most frequently was the generalized static stability criterion: that the coefficient of the zeroth power of the eigenvalue in the characteristic equation be greater than zero. In general, this criterion leads to $C_{n_{\beta_{\mathrm{dyn}}}}$ for the open-loop system and $C_{n_{\beta_{\mathrm{apparent}}}}$ for the closed-loop system, as well as to the LCPD and ARI requirements for selected feedback gain structures.

In obtaining these results, several assumptions were made and, in addition, a considerable amount of information was not used that is provided by the remaining criteria of Routh. Included in this category are that the remaining coefficients of the characteristic equation must be greater than zero and that each of the Routh discriminants must be greater than zero. These additional criteria provide information on dynamic stability requirements, as well as static stability requirements.

The idea of Pelikan,[16] which uses secant derivatives, can be used to extend results obtained from linear system analysis to be applicable to nonlinear systems. In addition, the direct application of the basic idea of restoring moments or accelerations generated by perturbations in the variables was shown to lead to the same results as the linear theory provided the moment of interest was measured about the axis along which the angular perturbation displacement was taken.

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